

Non-Bunch-Davis Initial State Reconciles Chaotic Models with BICEP and Planck

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The BICEP2 experiment has announced a signal for primordial gravity waves with tensor-to-scalar ratio $r = 0.2^{+0.07}_{-0.05}$ [1]. This result may be reconciled with the Planck experiment [2] if there is a considerable tilt of r , \mathcal{T}_r , with a positive sign, $\mathcal{T}_r = d \ln r / d \ln k \sim 0.16$ corresponding to a blue tilt for the tensor modes of order $n_T \simeq 0.12$. Simple slow-roll models fail to provide this tilt [1]. In this note we show that a non-Bunch-Davis initial state for perturbations not only provides a match between large field chaotic models (like $m^2 \phi^2$) with the Planck [3], but also naturally accommodates the running of the tilt needed to explain the BICEP2 result.

Early Universe cosmology has become a very active area of research in the last decade or so, as there is a wealth of precise cosmic microwave background (CMB) measurements pouring in. In particular, since last year two major collaborations Planck [2] and BICEP [1] have announced their results. The CMB measurements analyzed with other cosmological data favor the simple Λ CDM model for late time cosmology and inflationary paradigm for early stages of Universe evolution. According to the Planck collaboration data [2] the power spectrum of CMB temperature fluctuations (or as it is known, the power spectrum of curvature perturbations) \mathcal{P}_S is measured to be about 2.195×10^{-9} . The spectrum is almost flat, with a few-percent tilt toward larger scales (i.e., red spectrum) and is almost Gaussian.

Planck took cosmologists by surprise as it not only did not observe non-Gaussianity, which could have been used to considerably constrain inflationary models, but also put a strong upper bound on the amplitude of primordial gravity waves during inflation. These gravity waves are tensor mode fluctuations which are produced during inflation. The power spectrum of gravity waves \mathcal{P}_T is usually reported through the tensor-to-scalar ratio $r = \mathcal{P}_T / \mathcal{P}_S$ which Planck reported to be bounded at 2σ level as $r < 0.12$. This bound corresponds to the pivot scale $k_* = 0.002 Mpc^{-1}$. The tilt in the power spectrum of curvature perturbations is customarily denoted by $n_S - 1$, $n_S - 1 = d \ln \mathcal{P}_S / d \ln k$, where k is inverse of the scale. Planck constrained $n_S - 1 = -0.0397 \pm 0.0146$ at 2σ level. Planck's measurement already disfavored many single field models, especially those with convex potential [2].

CMB besides having one-in- 10^5 part temperature fluctuations is partially polarized and the parity odd polarization, the B-mode, is usually attributed to primordial gravity waves, tensor modes [4]. BICEP2 collaboration has recently announced observation of B-mode polarization [1]. BICEP results took cosmologists by an even

greater surprise, when measured $r = 0.2^{+0.07}_{-0.05}$. This was not an outright inconsistency between the two collaborations though, because BICEP focused on smaller scales than Planck announced bound; BICEP data is for $\ell \sim 80$. BICEP result was challenging in view of Planck results, as the measured value is already in the region which was excluded by Planck, unless the power spectrum of gravity waves considerably grows as we move to smaller scales, i.e. a blue, with relatively large tilt, for power spectrum of tensor modes. This is one possibility to reconcile BICEP data with Planck results [1]. Nonetheless, this potential way for Planck-BICEP reconciliation seems very hard to achieve in the context of slow-roll inflationary models composed of scalar fields minimally coupled to Einstein gravity. To see this, we need to go through the equations more closely.

The controversy is best formulated in terms of the tilt of tensor-to-scalar ratio \mathcal{T}_r ,

$$\mathcal{T}_r \equiv \frac{d \ln r}{d \ln k} = \frac{d \ln \mathcal{P}_T}{d \ln k} - \frac{d \ln \mathcal{P}_S}{d \ln k} = n_T - (n_S - 1), \quad (1)$$

where n_T is the tilt of power spectrum of tensor modes and $n_S - 1$ is the tilt of the power spectrum of curvature perturbations. Planck requires $n_S - 1$ to be negative and of order -0.04 . Standard, textbook analysis for slow-roll inflationary models leads to the “consistency relation” $n_T = -r/8$ [5], which is a red-tilt for gravity waves [6, 7]. Therefore, n_T , too, is negative and of order $\mathcal{O}(-0.01)$ for such inflationary models. On the other hand, BICEP-Planck reconciliation requires

$$\mathcal{T}_r \geq +0.16. \quad (2)$$

This clearly shows the tension between standard slow-roll models, and in particular the consistency relation with Planck+BICEP data: Slow-roll inflationary models cannot easily and readily accommodate the respectively large value of tensor-to-scalar spectral tilt \mathcal{T}_r and the blue

tensor spectrum required by recent observations (please see [8] for another attempt to make r run).

As discussed, in particular noting (1), to remedy Planck-BICEP tension we need to relax the consistency relation $n_T = -r/8$. The possibility which we will entertain here is based on the fact that in deriving standard cosmic perturbation theory results, besides the action of the model (which establishes the background inflationary dynamics and provides the equation of motion for cosmic perturbation fields), we also need to specify the initial quantum state over which these (quantum) cosmic perturbations have been produced. The standard initial state used is the Bunch-Davis (BD) vacuum state [9], stating that perturbation modes with physical momenta much larger than the Hubble scale during inflation H , effectively propagate in a vacuum state associated with flat space, the standard quantum field theory vacuum state.

Considering non-Bunch-Davis (non-BD) initial state for cosmic perturbations during inflation provides the setup to relax the consistency relation [10] (see [11] for some earlier works on the non-BD inflationary cosmology.) In fact, in our previous paper [3] we discussed such a setup and already used it in resolving the tension between Planck data and large-field chaotic inflationary models, including the simplest inflationary model with $m^2\phi^2$ potential for the inflaton field ϕ . Large-field models generically predict large value for tensor-to-scalar ratio r , with $r \sim 0.05 - 0.2$ [12]. So, they are potentially very good candidates for accommodating BICEP too. As we will discuss here, non-BD initial state can equip the large-field models with the tilt of r , \mathcal{T}_r , (equivalent with blue tensor spectrum, $n_T > 0$) needed for BICEP-Planck reconciliation; the chaotic model $m^2\phi^2$ [12] with non-BD initial state nicely fits with all available cosmological data.

The rest of this Letter is organized as follows. We first briefly review the setup presented in [3] to fix our notations. We then show that a mild tilt in the non-BD initial state will accommodate BICEP as well as Planck data. In the end we make some concluding remarks.

Power spectra and non-BD initial state. Here we consider a simple single-field inflationary model described by the action

$$\mathcal{L} = -\frac{M_{\text{pl}}^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi), \quad (3)$$

where $M_{\text{pl}} = (8\pi G_N)^{-1/2} = 2.43 \times 10^{18}$ GeV is the reduced Planck mass. We take our model to be a chaotic inflation large-field model [6], motivated by the recent observation of tensor modes [1], e.g. $V(\phi) = \frac{1}{2}m^2\phi^2$. The details of cosmic perturbation theory analysis for this model in standard Bunch-Davis vacuum may be found in standard textbooks, e.g. [6], and the modifications due to non-BD initial state in [3, 11]. For completeness we have gathered a summary of this analysis in the appendix. The power spectra and tensor-to-scalar ratio, r ,

are

$$\begin{aligned} \mathcal{P}_S &= \frac{1}{8\pi^2\epsilon} \left(\frac{H}{M_{\text{pl}}} \right)^2 \gamma_S \\ \mathcal{P}_T &= \frac{2}{\pi^2} \left(\frac{H}{M_{\text{pl}}} \right)^2 \gamma_T \\ r &= \frac{\mathcal{P}_T}{\mathcal{P}_S} = 16\epsilon \gamma, \end{aligned} \quad (4)$$

with

$$\gamma_S = |\alpha_k^S - \beta_k^S|^2_{k=\mathcal{H}}, \quad \gamma_T = |\alpha_k^T - \beta_k^T|^2_{k=\mathcal{H}}, \quad \gamma = \frac{\gamma_T}{\gamma_S}, \quad (5)$$

where α 's and β 's parameterize non-BD initial state for scalar and tensor modes and the spectral tilts are then¹

$$\begin{aligned} n_S - 1 &= (n_S - 1)_{\text{BD}} + \frac{d \ln \gamma_S}{d \ln k}, \\ n_T &= (n_T)_{\text{BD}} + \frac{d \ln \gamma_T}{d \ln k}, \\ \mathcal{T}_r &= (\mathcal{T}_r)_{\text{BD}} + \frac{d \ln \gamma_T}{d \ln k} - \frac{d \ln \gamma_S}{d \ln k}. \end{aligned} \quad (8)$$

The Lyth bound [13] and the consistency relation will also be modified due to the non-BD effects to [3]

$$r \lesssim 2.5 \times 10^{-3} \left(\frac{\Delta\phi}{M_{\text{pl}}} \right)^2 \gamma, \quad r = -8n_T\gamma, \quad (9)$$

where $\Delta\phi$ is the inflaton field displacement during inflation. The modification in the consistency relation is, as discussed, what can resolve the mismatch of slow-roll models with BICEP+Planck data.²

Parameterizing the initial states. Noting the normalization conditions (25) and (29) and that only the phase difference between α_k and β_k matters, the non-BD initial states may be parameterized as [3]

$$\begin{aligned} \alpha_k^S &= \cosh \chi_S e^{i\varphi_S}, & \beta_k^S &= \sinh \chi_S e^{-i\varphi_S} \\ \alpha_k^T &= \cosh \chi_T e^{i\varphi_T}, & \beta_k^T &= \sinh \chi_T e^{-i\varphi_T}. \end{aligned} \quad (10)$$

We consider a crude model in which [14],

$$|\beta_k^{\{S,T\}}| \propto \beta_0^{\{S,T\}} \exp \left\{ -k^2 / [Ma(\tau)]^2 \right\} \quad (11)$$

¹ It is instructive to note and recall expressions for the tilts of power spectra and scalar-to-tensor ratio r for $\lambda\phi^n$ chaotic models in the BD vacuum. For these models $\eta = 2(n-1)\epsilon/n$, and

$$(\mathcal{T}_r)_{\text{BD}} = +\frac{4}{n}\epsilon, \quad (n_S - 1)_{\text{BD}} = -\frac{2(n+2)}{n}\epsilon. \quad (6)$$

Noting that $r \propto \epsilon \propto (n_S - 1)$, one can relate the tilt of r to the running of the spectral tilt ξ . Explicitly,

$$(\mathcal{T}_r)_{\text{BD}} = \frac{\ln(1 - n_S)}{d \ln k} = \frac{1}{n_S - 1} \frac{dn_S}{d \ln k} = \frac{1}{n_S - 1} \xi. \quad (7)$$

² As we discussed in [3], in major part of the constrained non-BD parameter space, $\gamma \leq 1$ and effective field theory could not be saved by reducing $\Delta\phi < M_{\text{pl}}$, enhancing γ .

(or any smooth function in which $|\beta_k|^2$ falls off as $k^{-(4+\delta)}$). Here M is a super-Hubble energy scale associated with the new physics which leads to the non-BD initial state. In this scenario, all the k modes are pumped to an excited state as their physical momentum reaches the cutoff $\frac{k}{a(\tau)} = M$. The choice in (11) indicates that M is the (cutoff) scale at which the mode gets excited from Bunch-Davies vacuum.

The physically allowed region in the four parameter space of initial states is subject to the following constraints: (1) Absence of backreaction of initial states on the inflationary background; (2) Planck normalization for \mathcal{P}_S ; (3) value of spectral tilt $n_S - 1$ as observed by Planck; (4) fitting the value of r and the corresponding tilt \mathcal{T}_r , as required by BICEP+Planck, *i.e.* we take $r_{Planck} \leq 0.12$ (at $k_* = 0.002 Mpc^{-1}$ and $r_{BICEP} \simeq 0.2$ (at $\ell \sim 80$)). In our analysis we focus on large-field single-field models. The first three conditions were also considered in [3] while the fourth one is new.

Absence of backreaction of initial excited state on the background slow-roll inflation trajectory implies that the energy stored in the initial non-BD state for both scalar and tensor sectors should not exceed the change in the energy density in one e-fold. This condition is fulfilled if [3]

$$\sinh \chi_S \lesssim \epsilon \frac{H M_{Pl}}{M^2}, \quad \sinh \chi_T \lesssim \epsilon \frac{H M_{Pl}}{M^2}. \quad (12)$$

The above indicates that the upper bound on the deviation from BD initial state measured by χ_S is inversely proportional to the scale of new physics M . Hence, larger values of M require smaller χ_S . The COBE normalization implies

$$\frac{H}{M_{Pl}} = \frac{1}{\sqrt{\gamma_S}} 3.78 \times 10^{-5}. \quad (13)$$

Assuming n_S takes its best fit value of Planck, $n_S - 1 \simeq -0.04$, and that $\epsilon \sim 0.01$, then $d \ln \gamma_S / d \ln k \lesssim 10^{-2}$.

The above conditions are achieved if we take χ_T and χ_S to take typical values [3], *i.e.* $\sinh \chi_S \simeq e^{\chi_S} / 2$, $\sinh \chi_T \simeq e^{\chi_T} / 2$ and hence

$$\gamma_S \simeq e^{2\chi_S} \sin^2 \varphi_S, \quad \gamma_T \simeq e^{2\chi_T} \sin^2 \varphi_T.$$

Moreover to be able to rely the effective field theory methods, we are typically interested in larger values of M which is possible if φ_S is close to maximal; $M \simeq 20H$ happens when $\varphi_S \sim \pi/2$ [3].

To reconcile BICEP+Planck we want $n_S - 1 \sim -0.04$ and $\mathcal{T}_r \geq +0.16$ and the Planck bound on r requires $\gamma < 3/4$. Therefore,

$$\begin{aligned} e^{2(\chi_T - \chi_S)} \sin^2 \varphi_T &< 3/4, \\ \frac{d\chi_S}{d \ln k} &\leq 10^{-2}, \\ \frac{d\chi_T}{d \ln k} + \cot \varphi_T \frac{d\varphi_T}{d \ln k} &\gtrsim 0.08. \end{aligned} \quad (14)$$

We need not impose any condition on $\frac{d\varphi_S}{d \ln k}$, as $\partial \ln \gamma_S / \partial \varphi_S = 0$ at $\varphi_S = \frac{\pi}{2}$. Above we have also assumed that $\tan \varphi_T \gg e^{-2\chi_T}$. If $\chi_T \gtrsim 1$, in principle very small values for φ_T could be achieved.

One interesting example that is compatible with the above constraints is when $\chi_T = \chi_S$. This would correspond to the case where the numbers of particles in the tensor and scalar excited states are equal. Change in $n_S - 1$ from its Bunch-Davies value could be set to zero, if $d\chi_S / d \ln k = 0$. Since $\chi_S = \chi_T$, one has to assume that χ_T is scale independent too. A positive tensor spectral index would come totally from the scale-dependence of φ_T . The amount of suppression of $r_{0.002}$, will be equal to $\sin^2 \varphi_T$. As for $\chi_T \gtrsim \text{few}$ very small φ_T 's are approachable, this would correspond to the case where the tensor and scalar modes phases are complementary angles (roughly $\varphi_S + \varphi_T = \pi/2$) when they cross the scale of new physics M . For example, for $\chi_T \gtrsim 2$

$$\varphi_T \sim 0.1, \quad \frac{d\varphi_T}{d \ln k} \gtrsim 0.01, \quad (15)$$

would be enough to solve the tension between Planck and BICEP2 data.

Another interesting option is when r for our chaotic model is reduced just enough to reconcile with the Planck data. For $m^2 \phi^2$, this would mean a factor of 3/4 of its Bunch-Davies value. The corresponding φ_T for such a scenario is

$$\varphi_T \simeq 1.04. \quad (16)$$

The variation of φ_T with scale has to be

$$\frac{d\varphi_T}{d \ln k} \gtrsim 0.14 \quad (17)$$

For such a scenario, around $k \simeq 50 Mpc^{-1}$, the tilt of the tensor spectrum becomes zero and the spectrum becomes red again for larger k 's.

The other possibility to obtain positive \mathcal{T}_r and hence a blue tensor spectrum is to allow for running of χ_T . If this is the sheer cause of a blue gravitational spectrum a value of

$$\frac{d\chi_T}{d \ln k} \gtrsim 0.08 \quad (18)$$

is required to solve the discrepancy between BICEP and Planck data. Depending on the value of φ_T , one has to ensure the required suppression through the γ factor.

CONCLUDING REMARKS

As discussed in [3] non-Bunch-Davis initial condition for inflationary perturbations provide, with a typical value of the χ_S parameter ($\chi_S \gtrsim 1$) with the non-BD phase φ_S close to maximum, $\varphi_S \sim \pi/2$, can reconcile the

$m^2\phi^2$ chaotic model with Planck data, $r_{0.002} < 0.12$, if the scale of new physics which sources the non-BD initial state M , is around $20H$. Observation of B-modes by the BICEP experiment at $\ell \simeq 80$ can be matched with the bound from Planck data, if the gravity wave spectrum has a blue tilt of order 0.12.

Slow-roll inflation with BD initial condition cannot provide such a blue tilt. One can obtain such a blue spectrum if the tensor Bogoliubov coefficient has a small phase ($\varphi_T \lesssim 0.1$) with small k -dependence, $\partial\varphi_T/\partial\ln k \sim 0.01$. Simple chaotic models, in particular $m^2\phi^2$ model, have been of interest because they are endowed with simplicity and beauty. As our analysis indicates they can be compatible with both Planck and BICEP results, if perturbations start in a non-BD initial state at the beginning of inflation. Noting that B-modes are coming from purely tensor perturbations of the metric [15] and that initial non-BD for perturbations is provided from a high energy pre-inflationary physics, such resolutions may open up a window to the realm of quantum gravity, a territory which is untouchable by collider experiments.

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Cosmological perturbations in non-BD initial state

To explore the effects of non-Bunch-Davis (non-BD) initial state of perturbations and setup our notations, we briefly review cosmic perturbation theory. More detailed analysis in standard BD vacuum may be found in many textbooks e.g. [6], more detailed discussion on non-BD may be found in [3] and references therein. Here we will consider slow-roll models described by the action (3).

The space-time metric in presence of scalar and tensor perturbations can be parameterized as

$$ds^2 = a^2(\tau) \left[-(1 + 2\Phi)d\tau^2 + ((1 - 2\Psi)\delta_{ij} + h_{ij}) dy^i dy^j \right].$$

Φ and Ψ are the scalar Bardeen potentials which are equal for the scalar-driven inflationary model we are considering. h_{ij} is a symmetric divergence-free traceless tensor field, $h_i^i = 0$, $\partial^i h_{ij} = 0$. The inflaton field also fluctuates around its homogeneous background value

$$\phi(\tau) = \phi_{\text{hom.}}(\tau) + \delta\phi. \quad (19)$$

where $\phi_{\text{hom.}}(\tau)$ is the homogeneous part of the inflation which satisfies $\delta\phi \ll \phi_{\text{hom.}}(\tau)$. For the slow-roll a quasi-

de-Sitter inflationary trajectories

$$a(\tau) \simeq -\frac{1}{H\tau} \quad (20)$$

$$\epsilon \equiv 1 - \frac{\mathcal{H}'}{\mathcal{H}^2} \ll 1, \quad \eta \equiv \epsilon - \frac{\epsilon'}{2\mathcal{H}\epsilon} \ll 1, \quad (21)$$

where H is the Hubble parameter during inflation and prime denotes derivative w.r.t. the conformal time τ .

Equation of motion for scalar perturbations, the gauge-invariant Mukhanov-Sasaki variable $u(\tau, y)$,

$$u = -z \left(\frac{a'}{a} \frac{\delta\phi}{\phi'} + \Psi \right), \quad z \equiv \frac{a\phi'}{\mathcal{H}}, \quad \mathcal{H} \equiv \frac{a'}{a}, \quad (22)$$

is

$$u_k'' + \left(k^2 - \frac{z''}{z} \right) u_k = 0, \quad (23)$$

$u_k(\tau)$ is the Fourier mode of $u(\tau, y)$. The most generic solution to (23) in the leading order in slow-roll parameters ϵ, η may be expressed as:

$$u_k(\eta) \simeq \frac{\sqrt{\pi|\tau|}}{2} \left[\alpha_k^S H_{3/2}^{(1)}(k|\tau|) + \beta_k^S H_{3/2}^{(2)}(k|\tau|) \right], \quad (24)$$

where $H_{3/2}^{(1)}$ and $H_{3/2}^{(2)}$ are respectively Hankel functions of the first and second kind. The coefficients α_k^S and β_k^S are in general scale-dependent and may have non-trivial scale-dependent phases. They respectively behave like the positive and negative frequency modes. These Bogoliubov coefficients satisfy the normalization condition

$$|\alpha_k^S(k)|^2 - |\beta_k^S(k)|^2 = 1. \quad (25)$$

The standard BD vacuum corresponds to $\alpha_k = 1$ and $\beta_k = 0$. However, in general new physics at the onset of inflation can provide us with generic non-BD initial state parameterized with generic α_k^S and β_k^S . The power spectrum of curvature perturbations is

$$\mathcal{P}_S = \frac{k^3}{2\pi^2} \left| \frac{u_k}{z} \right|_{k/\mathcal{H} \rightarrow 0}^2. \quad (26)$$

which for simple chaotic slow-roll models reduce to

$$\mathcal{P}_S = \mathcal{P}_{BD} \gamma_S, \quad (27)$$

where

$$\mathcal{P}_{BD} = \frac{1}{8\pi^2\epsilon} \left(\frac{H}{M_{\text{pl}}} \right)^2, \quad \gamma_S = |\alpha_k^S - \beta_k^S|_{k=\mathcal{H}}^2. \quad (28)$$

Similarly, one may consider the tensor mode perturbations in a non-BD initial state parameterized by α_k^T and β_k^T subject to the normalization condition

$$|\alpha_k^T|^2 - |\beta_k^T|^2 = 1. \quad (29)$$

The power spectrum of tensor modes is then given by [3]

$$\mathcal{P}_T = \mathcal{P}_{BD}^T \gamma_T, \quad (30)$$

where

$$\mathcal{P}_{BD}^T = \frac{2}{\pi^2} \left(\frac{H}{M_{\text{pl}}} \right)^2, \quad \gamma_T = |\alpha_k^T - \beta_k^T|_{k=\gamma_{\mathcal{C}}}^2. \quad (31)$$

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